INVERSE PROBLEM OF HEAT CONDUCTION WITH ACCOUNT OF THE TEMPERATURE DEPENDENCE OF THE THERMOPHYSICAL CHARACTERISTICS

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A method for solving a nonlinear inverse problem of heat conduction on electrical models is stated.

As is known, for account of the temperature dependence of thermophysical characteristics, the heat-conduction problem becomes nonlinear and special means are required for solving it. This also refers to the inverse heat-conduction problem.

In [1] methods for the solution of the direct heat-conduction problem in a nonlinear formulation on electrical models were discussed. If we use the same method as in [1] for solving the inverse problem, i.e., if we linearize the heat-conduction equation with the help of transformations of the type of a Kirchhoff substitution

$$\theta = \int_0^T \lambda(T)\, dT, \tag{1}$$

Shneider substitution

$$\theta = [\lambda(T)]^2 \tag{2}$$

etc., then the main complication will be in the realization of nonlinear boundary conditions of the third kind

$$\alpha [T_c - T_s(\theta)] = -\frac{\partial\theta}{\partial n}, \tag{3}$$

where the form of the function $T_s(\theta)$ is determined by the form of the function $\lambda = f(T)$.

Unlike devices for the solution of direct heat-conduction problems, the assemblies for the realization of the boundary conditions for an inverse problem are more complicated, since the unknown quantity (the coefficient α) should be determined, precisely on them, and not on a passive model, which is free from nonlinearities, as occurs in the solution of the direct heat-conduction problem.

On the basis of the devices developed by us for solving inverse-problems in the nonlinear formulation, just as for the case of direct problems, we assume methods of nonlinear resistances and combined circuits.

Below we present the circuit of the device that the method of nonlinear resistances is based on. According to this method [1], the thermophysical nonlinearities are modelled using nonlinear electrical elements, whose volt–ampere characteristics have a form similar to the nonlinearity being modelled. Since the experiment shows that the basic form of the nonlinearity in the problems being solved is a parabolic nonlinearity, it seems advisable that we use as nonlinear elements electron multielectrode tubes of pentode type, the anode characteristics of which represent a family of curves of parabolic type

$$I = Au^n \tag{4}$$

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with different coefficients A and exponents n. The value of n depends on the value of the resistances inserted in parallel and in series with the tube. As for the coefficient A, its value is determined mainly by the bias on the control grid of the tube. As nonlinear resistances we can also use semiconducting elements; however, here we discuss only the tube circuit, since a transistor device is still in the development stage.

The choice of the tube regime depends on the conditions of the problem. Thus, the exponent n is entirely determined by the form of the function $\lambda = f(T)$. For example, for the case of a linear temperature dependence of the thermal-conductivity coefficient, the function $\theta = f(T)$ proves to be quadratic, and the exponent $n = 1/2$. The coefficient A is the analog of the heat-exchange coefficient α, and, hence, the latter on the electrical model can be given by the bias on the control grid of the tube. This circumstance is used in producing the device for solving the inverse heat-conduction problem, which is the question that is considered below.

Unlike the devices for assigning the nonlinear boundary conditions for the solution of the direct heat-conduction problem, when the nonlinearity of the anode characteristics of the tube was used for modelling only the nonlinear term (3), and the linear term on the model was realized by using a current regulator, in the present case, both terms on the left side of Eq. (3) are modelled by using tubes. This is due to the fact that here the unknown is the heat-exchange coefficient, which enters into both terms on the left side of Eq. (3).

Without loss of generality, for simplicity we consider the case of a linear function $\lambda = f(T)$, when after using the Shneider formulation [Eq. (2)], we can write Eq. (3) in finite-difference form as

$$\alpha(\sqrt{\theta_c} - \sqrt{\theta_M}) = \frac{\theta_M - \theta_N}{h}, \tag{5}$$

where θ_M and θ_N are the values of the function θ at the boundary point and at a point a half-spacing away from the boundary point; h is the spacing of the grid.

If, to the boundary point of the passive model at which the temperature field of the body is modelled, we connect two identical tubes in the manner shown in Fig. 1, we set $V_0 = 0$, and the tubes are first adjusted to the $n = 0.5$ regime, then for the boundary point we can write the Kirchhoff law as

$$I_2 - I_1 = I_3$$

or

$$A(\sqrt{V_c} - \sqrt{V_M}) = \frac{V_M - V_N}{r}, \tag{6}$$

where V_c, V_M, and V_N are the potential-analog of the function θ_c, and the potential of the boundary point and a point a half-spacing away from the boundary point; r is the resistance between these points.

The bias on the control grids of the tubes and, hence, also the coefficients A in (6) are determined automatically as a result of the interaction between the mismatch signal and the remaining elements of the circuit (Fig. 1). This is accomplished as follows.

After connecting the pushbutton (PB), the initial grid bias U_1 is fed to the control grids of the tubes T1 and T2, the current proceeds to the boundary point of the passive model (PM), and a certain potential field is formed at the PM.

The signal from the node point of the PM is fed to the input of an adder – subtractor AS, at the second input of which is fed the potential from a voltage divider [voltage-divider potential] (VDP), corresponding to the function θ at the node point indicated above. From the output of the AS, the mismatch signal is fed to the input of the adder A-1, there it is added with the similar signals fed from the other AS's. The added mismatch signal U_m is fed to the input of the adder A-2, where it is added with the output signal U_{c1}, which is fed to the second input of A-2, which is fed back to the output of A-2.

At the output of A-2, the bias voltage U_{c1} is formed, which varies as long as the potentials at the node points of the model are not equal to the VDP's, corresponding to the function θ at these points. The voltmeter V, connected between the cathode of tube T1 and its control grid, shows the bias voltage, which with accuracy up to a constant factor gives the magnitude of the heat-exchange coefficient. If the voltmeter scale is calibrated taking into account the conversion factors in $W/m^2 \cdot deg$, then we can immediately obtain the value of α.

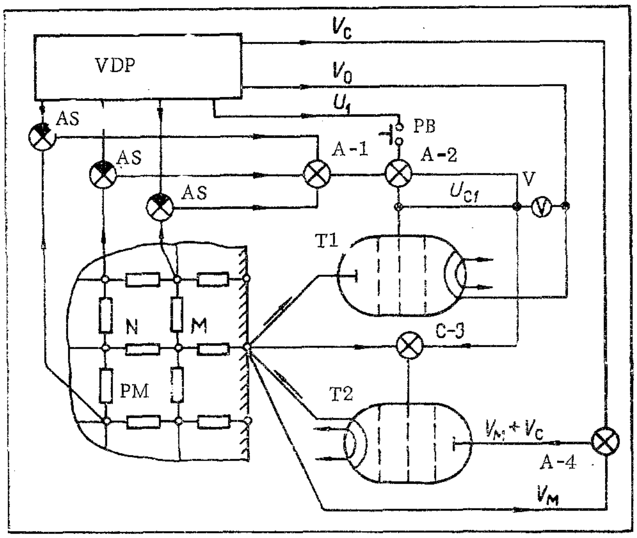

Fig. 1. Diagram illustrating the solution of the problem by the method of nonlinear resistances.

In order for the current passing through tube T2 to be equal to $I_2 \sim \sqrt{V_c}$, its cathode is connected to the node point, and to the anode there is fed the added signal $(V_M + V_c)$ from the output of the adder A-4, the input of which is connected with the VDP and the boundary point of the model. The coefficient A at the anode characteristic $I_2 = A\sqrt{V_c}$ of tube T2 proves to be equal to the analogous coefficient in the characteristic of the tube T1, since the bias voltages on the control grids of both tubes with respect to their respective cathodes prove to be equal. This results from the fact that on the grid of tube T2 there is fed a voltage from adder A-3,where the voltage U_{c1} is added to the potential of the boundary node point.

We note that the proposed circuit can be used to solve both linear and nonlinear problems to the same extent. Above, we develop the more complicated case, when the temperature dependence of the thermal-conductivity coefficient is taken into account. In the linear case, when it is necessary that the boundary conditions

$$\alpha (T_c - T_s) = -\lambda \frac{\partial T}{\partial n} \tag{7}$$

be satisfied on the model, we can use the linear sections of the anode characteristics of the multielectrode tubes or the control linear resistances, triodes, etc.

Transforming to the method of combined circuits, we recall that, in addition to passive models, it assumes the use of assemblies that operate according to the principle of electrical modelling.

The proposed device (Fig. 2), in addition to the system operating the added mismatch signal, which is entirely similar to that already considered, includes an adder A-2, an adder – subtractor AS-2, a multiplication unit MU, a control current regulator CR, and a functional transformer FT.

The device operates in the following manner. After the pushbutton PB is connected at the input of the adder A-2 and then to the multiplication unit MU there is fed a potential proportional to a certain initial value α_0. The added mismatch signal from the adder A-1 is fed to the input of the adder A-2, where it is added with the signal α, which is fed back from the output of A-2. The latter is also fed to the input of the multiplication unit MU, where it is multiplied by the output potential of AS-2. The inputs of AS-2 are connected with the VDP and through the functional transformer FT with the boundary point of the model. The output signal of the multiplication unit is fed to the input of the current regulator, where it is transformed into a current.

If the input signal is transformed in the functional transformer according to a law of inverse proportionality to the function $T = f(\theta)$, then at the output of the current regulator CR, the current formed will be proportional to the left side of Eq. (3).

The value of the heat-exchange coefficient in the appropriate scale is measured by a measuring device at the output of the adder A-2.

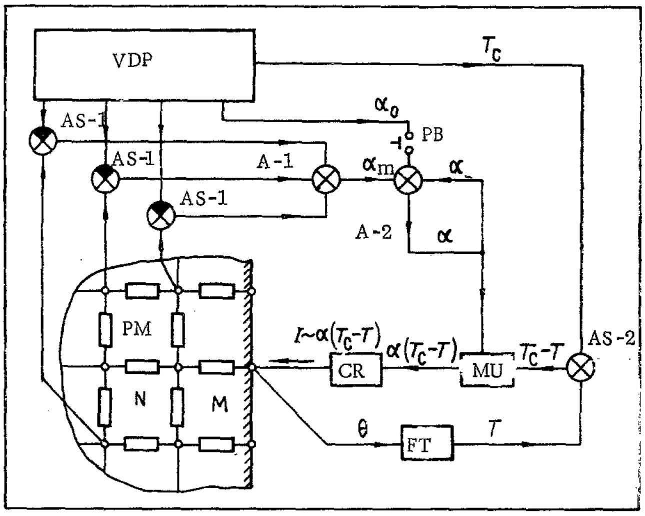

Fig. 2. Device for determining the heat-exchange coefficient (method of combined circuits).

Equation (3) can be modelled by an electrical model and also by the traditional method, i.e., when the analog of the thermal resistance serves as the active electrical resistance. However, in order to manage without a trial-and-error method, and to solve the problem in a single procedure, this resistance should be a control resistance. In [2] a follower system is proposed to control this resistance. Although it was intended for solving the linear problem, it can also be used to solve the inverse problem in a nonlinear formulation. Only the value of the heat-exchange coefficient will be determined not by the equation

$$\alpha = \frac{r\lambda}{Rh}, \tag{8}$$

as was the case in [2], but by the more complex function

$$\alpha = \frac{r\theta_c}{hRV_c} \frac{V_c - V_M}{T_c - T_M}. \tag{9}$$

In this expression the quantities r, θ_c, h, V_c, and T_c are known, and the remainder are determined by direct measurement.

Thus, the realization of the methods of nonlinear resistances and combined circuits using the devices described above makes it possible to solve the inverse heat-conduction problem in the nonlinear formulation, using standard circuits of passive models and electronic analog machines to do this.

NOTATION

T	is the temperature;
λ	is the thermal-conductivity coefficient;
α	is the heat-exchange coefficient;
I	is the electric current;
U	is the electric voltage;
R, r	are the electrical resistance;
V	is the electric potential;
h	is the spacing of grid.

Subscripts

s	denotes the surface;
c	denotes the medium and also grid;
M, N	denote the appropriate grid nodes;
m	denotes the mismatch.

LITERATURE CITED

1. Yu. M. Matsevityi, Solution of Nonlinear Problems of Technical Heat Physics Using Electrical Models, Author's Abstract of Doctor's Dissertation [in Russian], Minsk (1970).
2. Yu. M. Matsevityi, "Electrical modelling of an inverse heat-conduction problem," in: Heat and Mass Transfer [in Russian], Vol. 8, Nauka i Tekhnika, Minsk (1972).